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Research Article in Physics

Stability and Structural Limits of Tennis

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Abstract: A stable structure of tennis tower can be formed by stacking tennis balls to a certain structure. This paper will explore the structural stability of stacked tennis tower and the structural limit of the tennis tower and give the factors that limits the height growth of the tennis tower by experiments. It's proved that the friction coefficient of tennis surface is not a constant but gradually decreases with the increase of the pressure applied. Based on this conclusion and experimental data, it is deduced that the practical stacking limit of tennis tower is about 11 layers.

Keywords: Tennis tower; Friction; Structural limit

1. Introduction

Rogava from Ilia State University in Georgia reveals how simple friction allows bizarre towers to be built using tennis balls (Rogava.A, 2019). Firstly with 10 balls we can build a three-level pyramid with a triangular cross-section, which has 6 balls at the bottom, followed by 3 in the next layer and finally 1 ball on top. A tower can be made by carefully removing the three corner balls from the bottom layer of the pyramid. Interestingly, the corner balls in the second-bottom layer are kept in equilibrium, hanging over the layer below. These "exposed" balls are held in place because the balls directly above press down on them and into the two adjacent balls of the bottom layer – producing a pair of reaction forces to balance their weight. The torques are balanced too, with enough friction between the felt-covered balls to guarantee equilibrium. Moreover, this seven-ball structure can be made even higher by adding one extra three-ball layer after another, in which each tower has (3n + 1) balls, where n is the number of triangular layers. Experiments shows that as the tower goes taller, the difficulty to keep equilibrium grows. Therefore, we infer that there is a structural limit to the tennis tower.

Figure 1. Left was a tower made by the author, and right is one from Rogava (Rogava.A, 2019)

In this paper, we first derive the equilibrium conditions of the simplest three-layers tennis tower, and analyze the factors that determine the equilibrium of the tennis tower. Secondly, for tennis towers with more layers, the variation trend of the equilibrium condition is deduced, so as to find the parameters limiting the height growth of the tennis tower. Finally, according to experiments, the friction coefficient of the tennis ball skin is proved to change with the pressure, and the actual height limit size of the tennis ball tower is deduced.

2. Research Design, Data collection and analysis Methods

2.1. Stability of a three-layer tennis ball tower

Analysis: The conditions required for the static equilibrium of tennis balls are force balance and torque balance. For the three-dimensional structure in this example, choosing an appropriate method to optimize the calculation is the key to solving the problem. For this reason, we establish an orthogonal rectangular coordinate system as shown in Figure 2.

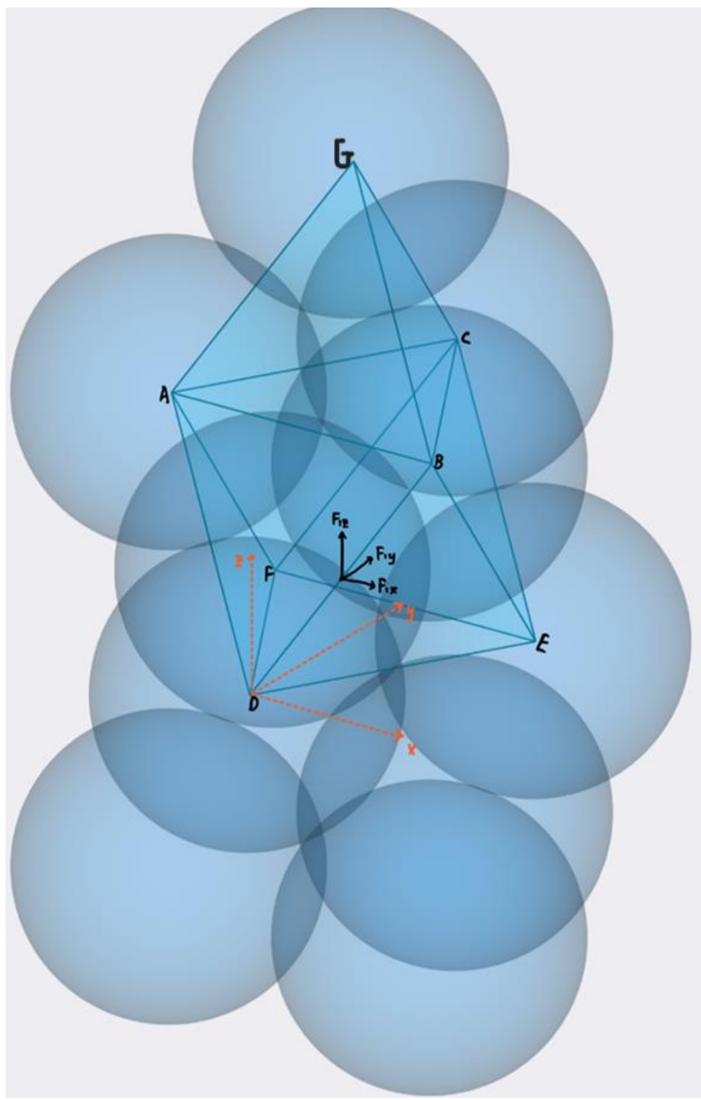

Figure 2. Three -dimensional structure of the three-layer tower

Solution: Project the three-layer tower composed of 7 balls to the horizontal plane. It is easy find that the tower has a symmetrical structure, based on which we can draw the horizontal force of the third-layer balls on the second-layer ones.

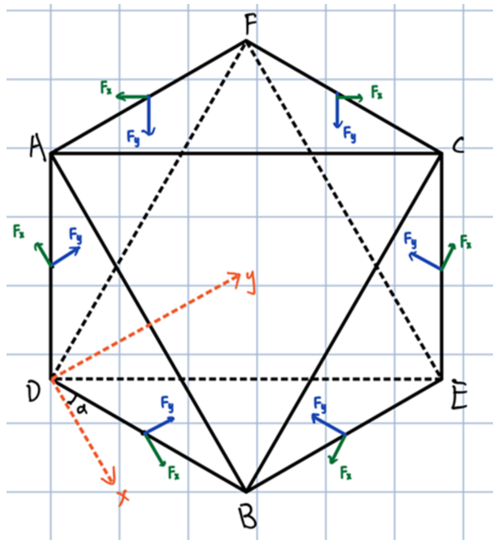

Figure 3. Planform of the tower and forces on each ball

To analyze the force on the tennis balls at each layer, we also need to project the tower from different sides, and simplify the three-dimensional problem into a two-dimensional one through its symmetrical structure.

2.1.1. Force on the top ball

In Figure 4, the top ball experiences gravitational force, with the normal force and the frictional force that the second layer act against it. According to the vertical force balance and the symmetrical structure of the top ball, we get:

$$N_0 \cos\theta + f_0 \sin\theta = \frac{1}{3}mg \tag{1}$$

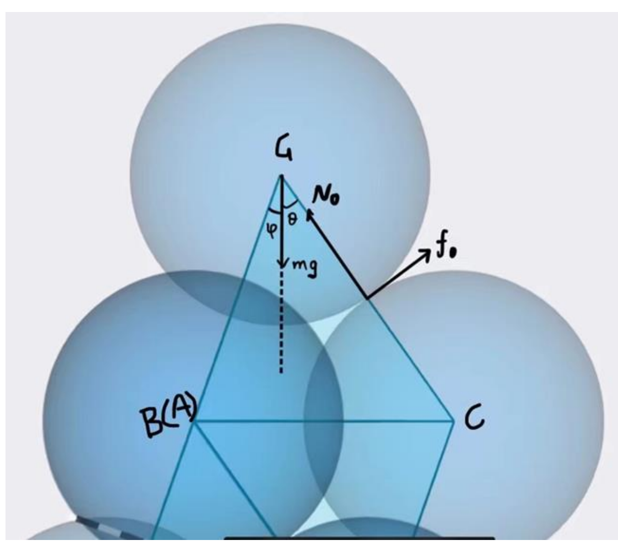

Figure 4. Force body diagram of the top ball observing from the left side

2.1.2. Force on the balls at the second layer

In Figure 5, the forces that acting on the second layer's ball are more complex. The second layer of the ball is squeezed in static state by the top and the third layer of the ball fixed. Based on the torque balance around the axis at the bottom of the balls at the second layer, we have:

$$\mathrm{N}_0 d\cos(\theta+\phi) + mgd\sin\phi = f_0(r + d\sin\theta) \tag{2}$$

Combining the above two equations, we can get:

$$\mathrm{f}_0 = \frac{\sqrt{3}}{6}mg \tag{3}$$

$$\mathrm{N}_0 = \frac{\sqrt{6}}{12}mg \tag{4}$$

$$\mu_{0\mathrm{Min}} = \frac{f_0}{N_0} = \sqrt{2} \tag{5}$$

Now, we have solved that the minimum static friction coefficient required for a three-layer tennis ball tower is $\sqrt{2}$.

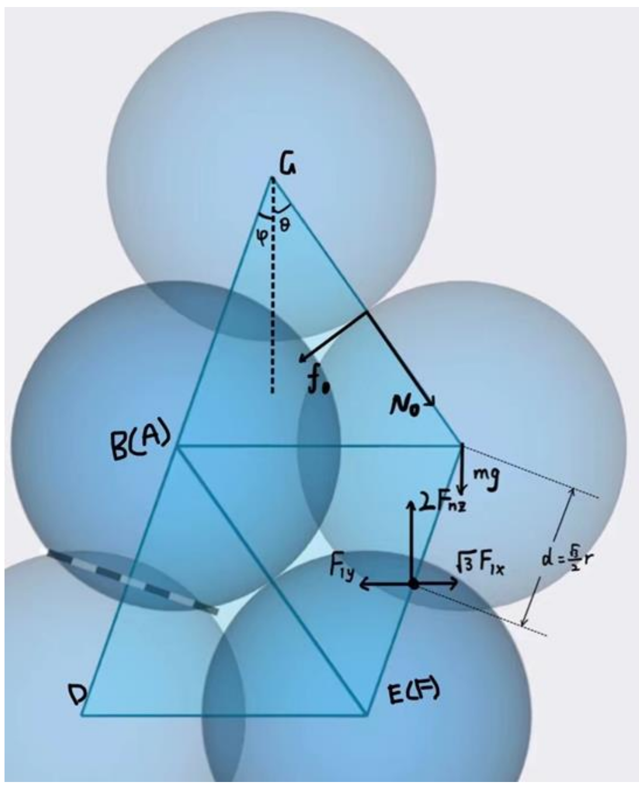

Figure 5. Force body diagram of the second layer's ball observing from the left side

2.2. Stability of tennis ball tower with arbitrary layers

Analysis: The next step is to find out the stability of the tower with arbitrary layers. When the layers can be set to a random number, the key to solving the problem is to find out the recursion formula of force between the layers, and thereby obtain the general solution.

Solution: The recurrence relation only starts from the third layer. Therefore, as a starting point, let us analyze the force on the balls at the third layer.

2.2.1. Solving the recursion formula

By the torque balance around point N, in Figure 6, we have:

$$\mathrm{mgd}\sin\phi + 2F_{(n-1)y}2d\cos\phi = 0 \tag{6}$$

So,

$$\mathrm{F}_{(n-1)y} = -\frac{\sqrt{2}}{16}mg \tag{7}$$

This proves that between the second layer and the second-to-bottom layer, F_y is a constant.

Then, from the horizontal force balance, we get:

$$2\mathrm{F}_{(n-1)y} + 2\mathrm{F}_{nx}\cos\alpha = 2F_{ny}\sin\alpha \tag{8}$$

So,

$$\mathrm{F}_{nx} = \frac{\sqrt{6}}{48}mg \tag{9}$$

From the above deduction, it can be concluded that between the second layer and second-to-bottom layer, the F_x is also a constant.

Based on the vertical force balance, we get:

$$2\mathrm{F}_{(n-1)z} + mg = 2F_{nz} \tag{10}$$

So,

$$\mathrm{F}_{nz} = F_{(n-1)z} + \frac{1}{2}mg \tag{11}$$

From this, we can see that between the second layer and the bottom layer, F_z is an arithmetic sequence.

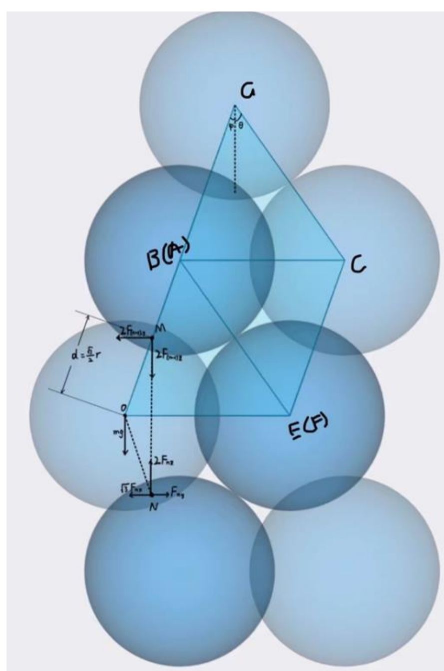

Figure 6. Force body diagram of the third layer's ball observing from the left side

2.2.2 The initial conditions for solving the sequence

With the above conclusions, according to the vertical force balance on the balls at the second laye, we can have:

$$\mathrm{mg} + N_0 \cos\theta + f_0 \sin\theta = 2F_{1z} \tag{12}$$

From the horizontal force balance, we get:

$$\mathrm{N}_0 \sin\phi - f_0 \cos\phi = 2F_{1y}\sin\alpha - 2\mathrm{F}_{1x}\cos\alpha \tag{13}$$

Combining the two equations, we can get the first term of the recursion sequence:

$$\mathrm{F}_{1z} = \frac{2}{3} mg \tag{14}$$

$$\mathrm{F}_{1x} = \frac{4\sqrt{2} - \sqrt{6}}{48} mg \tag{15}$$

The F_{1y} in the solution process is given by the torque balance equation of the balls at the third layer, which is a constant when the subscript is 1.

2.2.3 Solving the last term of the sequence

In Figure 7, the force on the balls at the bottom layer could be known.

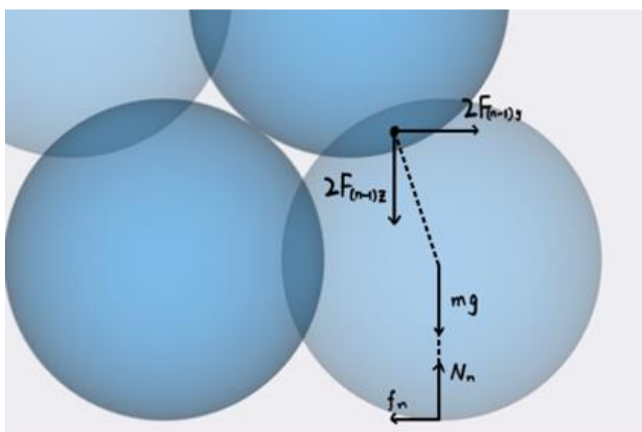

Figure 7. Force diagram of the bottom layer's ball observing from the left side

According to the torque balance around the touch down point, we get:

$$2\mathrm{F}_{(n-1)z} d \sin\phi = 2F_{(n-1)y}(r + d\cos\phi) \tag{16}$$

From the vertical force balance, we have:

$$2\mathrm{F}_{(n-1)z} + \mathrm{mg} = \mathrm{N}_n \tag{17}$$

Using the horizontal force balance, we get:

$$2F_{(n-1)y} = \mathrm{f}_n \tag{18}$$

Based on the general formula of the force in the z-direction, we can have:

$$\mathrm{F}_{(n-1)z} = \left(\frac{n-1}{2} + \frac{1}{6}\right) mg \tag{19}$$

The subscript n equals to the layer number minus 1. Therefore, the subscript of the bottom layer is defined as 0.

In this case, we get:

$$\mathrm{N_n} = \left(n + \frac{1}{3}\right) mg \tag{20}$$

$$\mathrm{F_{(n-1)y}} = \frac{\left(3-\sqrt{6}\right)}{3}\left(\frac{n}{2} - \frac{1}{3}\right) mg \tag{21}$$

$$\mathrm{f_n} = \frac{2\left(3-\sqrt{6}\right)}{3}\left(\frac{n}{2} - \frac{1}{3}\right) mg \tag{22}$$

Substituting the recursion formula of the force in the y-direction, we can have:

$$\mathrm{F_{(n-2)y}} = -\frac{\sqrt{2}}{16} mg \tag{23}$$

So,

$$\mathrm{F_{(n-1)x}} = \frac{\sqrt{3}}{3}\left(\frac{\left(3-\sqrt{6}\right)}{3}\left(\frac{n}{2} - \frac{1}{3}\right) + \frac{\sqrt{2}}{8}\right) mg \tag{24}$$

2.3 Layer limit of the tennis ball tower in practice

We have worked out the force at any contact point at any layer in the tower, thus the minimum static friction coefficient required for all contact surfaces can be solved. The approximate change trend can be obtained by computer simulation as follows:

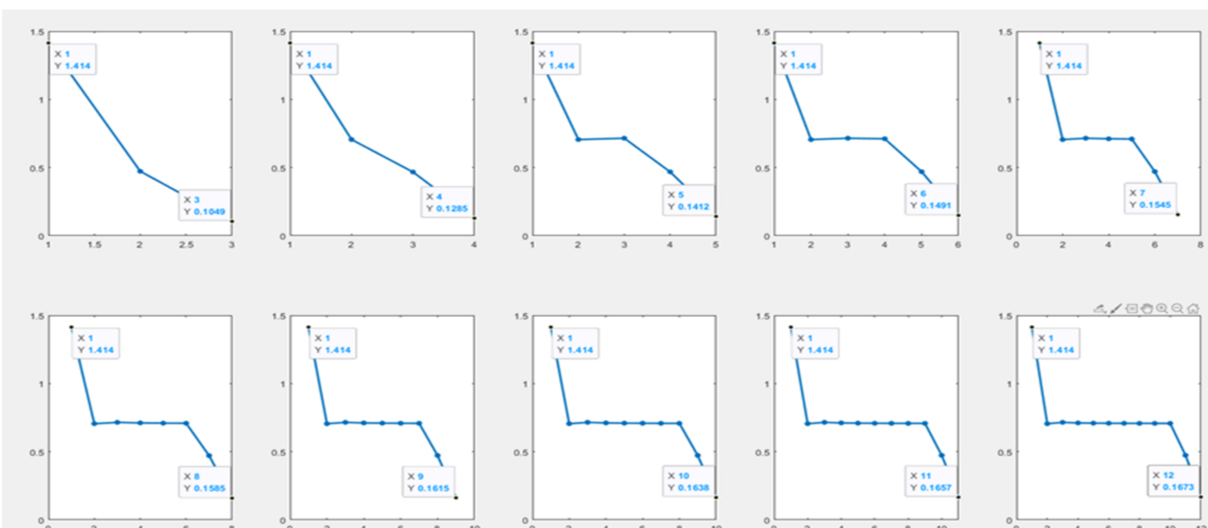

Figure 8. Friction coefficients that required for the balls from 3 to 12 layers

It can be seen from the above graph that the maximal static friction coefficient that the top ball requires is exactly 1.414, which is also the minimum static friction coefficient required by the three-layer tower.

Using deduction, it can be concluded that the top ball requires the highest static friction coefficient for equilibrium, which decreases gently as the layer goes down. The minimum static friction coefficient required between the balls at the bottom layer and the table takes another step downward.

Analysis: We can find through some simple experiments that the static friction coefficient between the ball surfaces is not always constant, but gradually decreases as the pressure at the contact point increases. The more layers, the heavier it is for the lower-layer balls and the higher static friction coefficient between the balls at lower layers due to the increasing pressure. This means that the actual static friction coefficient might be lower than the required one for the lower-layer balls.

To this end, we designed an experiment as follows.

Experiment design: Since the shape of tennis ball is a sphere, the measurement of friction coefficient needs to offset the torque. Therefore, we choose to disassemble a brand-new tennis ball to obtain the felt of the tennis ball. We spread the felt out into a flat surface and pasted it on the bottom of a lightweight carton, and another piece of felt to paste on an inclined plane.

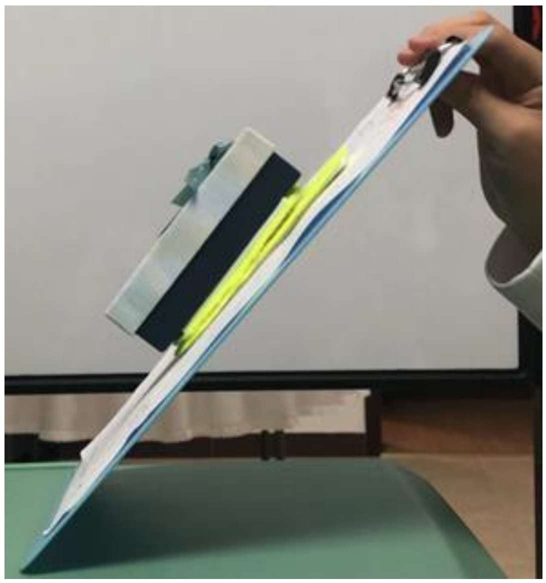

Figure 9. Equipment used in the frictional force test

We placed weights of different masses in the carton and place the carton on the inclined plane. The angle of the inclined plane was slowly raised until the carton just begins to slide. The weight of the weights in the carton, the angle of the inclined plane at the beginning of the slide and the weight of the carton itself were measured and recorded. The friction coefficient was obtained by the tangent of the angle of the inclined plane. The downward trend of static friction coefficient as shown in Figure 10.

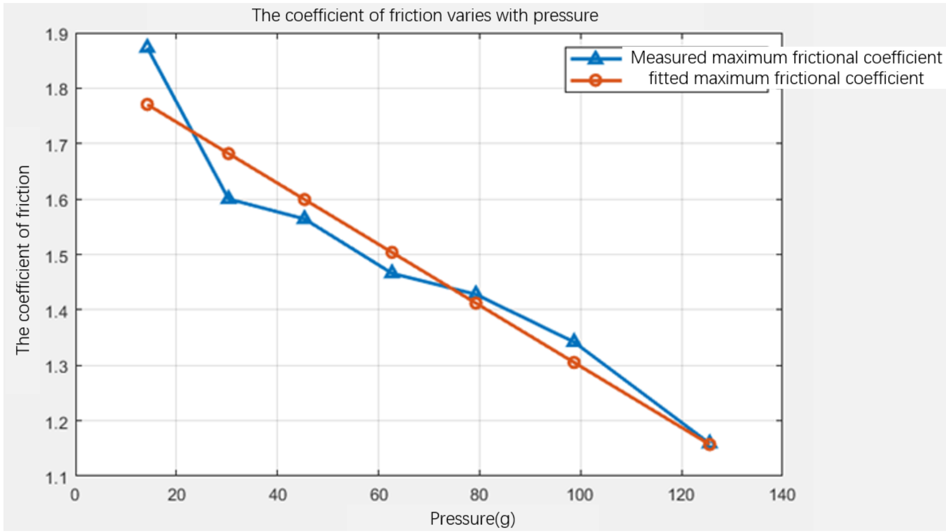

Figure 10. Coefficient of friction varies with pressure

Simulation: We use linear approximation to simulate the change of static friction coefficient and take a tennis ball weighing 57 grams as the subject. There are two lines in the graph below: the orange one represents the actual downward trend of the friction coefficient between the balls at each layer, while the blue one indicates the theoretical change trend of that required for the tower equilibrium.

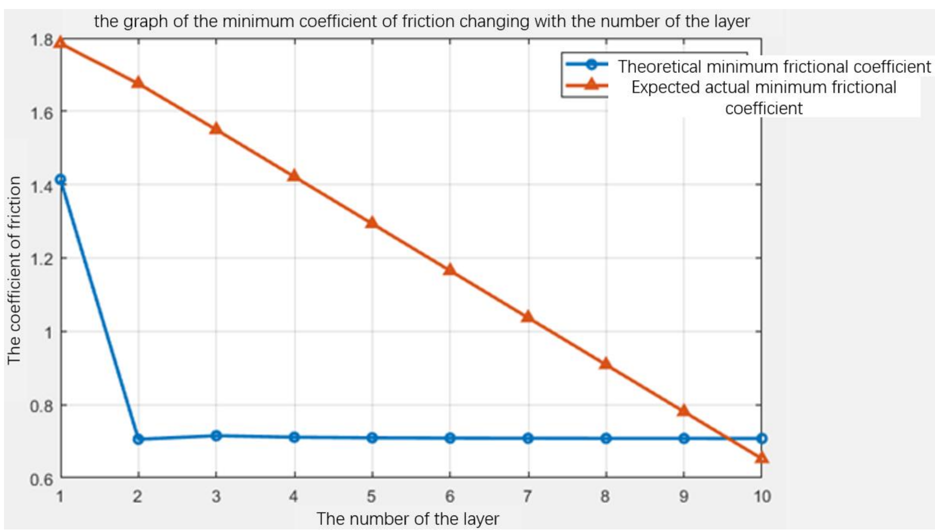

Figure 11. Minimum coefficient of friction changing with the number of the layer

Conclusion It can be seen from the Figure 11 that when the layer number exceeds 10, the actual friction coefficient between the tennis balls will be lower than the theoretical one. Therefore, it can be concluded that the layer limit of the tennis ball tower is around 10-11.

The current record for the highest tennis ball tower is made by Professor Andria Rogava from Ilia State University. The tower structure he built was first published in the magazine *Physics World* on May 23, 2019. The highest tower that he has successfully built so far is exactly of 10 layers, which verifies our theoretical and experimental results to a certain extent.

3. Results

3.1. Results of study 2.1

In section 2.1, we established the geometric model and decompose the forces in the system, and carried out the equilibrium analysis of the simplest three-layer tennis tower structure through the statics equilibrium conditions. It was concluded that the friction coefficient between the top layer and the middle layer of the three-layer tower is required to be at least greater than 1 for the balance of the tennis tower.

3.2. Results of study 2.2

In section 2.2, we selected two adjacent layers of a multi-layer tennis tower as the system, and deduced the recursive relationship of the normal force and friction force between the tennis balls as the number of layers changes according to the equilibrium condition, the general term formula can be given as:

$$\mathrm{F_{nz}} = F_{(n-1)z} + \frac{1}{2}mg \tag{11}$$

$$\mathrm{F_{(n-1)y}} = \frac{(3-\sqrt{6})}{3}\left(\frac{n}{2}-\frac{1}{3}\right)mg \tag{21}$$

$$\mathrm{F_{(n-1)x}} = \frac{\sqrt{3}}{3}\left(\frac{(3-\sqrt{6})}{3}\left(\frac{n}{2}-\frac{1}{3}\right)+\frac{\sqrt{2}}{8}\right)mg \tag{24}$$

$$\mathrm{f_n} = \frac{2(3-\sqrt{6})}{3}\left(\frac{n}{2}-\frac{1}{3}\right)mg \tag{22}$$

3.3 Results of study 2.3

Based on the conclusions derived in the previous two sections, an experiment was designed to measure the friction coefficient of tennis felt. The experimental results show that the friction coefficient of tennis felt decreases with the increase of the pressure applied on it.

With fitting the changing trend of friction coefficient of tennis felt with experimental data, we get the conclusion that the height limit of the actual tennis tower is about 10-11.

4. Conclusions and Synthesis

This paper first analyzes the structure of the tennis tower, and takes the simplest three-layer tennis tower as an example, obtains the size of the friction coefficient needed to keep the balance between the highest layer of tennis balls. Secondly, for the tennis tower with more layers, the relationship and changing trend of the friction coefficient required at each contact point are calculated layer by layer. The experiment shows that the friction coefficient of tennis felt is not a constant, but decreases with the increase of pressure. Therefore, the higher the tennis tower is, the more difficult it is to meet the condition of the bottom tennis ball to keep balance. According to the experimental data, it can be estimated that the height limit of the tennis tower is about 10-11 stories.

Because it is very difficult to build a tennis tower with higher height, we failed to verify the obtained limit of the tennis tower. However, the current record for the highest tennis ball tower is made by Rogava (Rogava.A, 2019). The highest tower that he has successfully built so far is exactly of 10 layers, which verifies our theoretical and experimental results to a certain extent.

Our vision for future experimental directions is to translate the difficulty of increasing the height of the tennis ball into increasing the weight of the tennis ball. Because the experiment proves that the friction coefficient is related to the pressure, and for a certain layer at the bottom, the change of the number of tennis layers on it equals to the change of the total weight of tennis balls. Therefore, our next experiment will fill the tennis balls with a certain amount of padding to verify our inference about the structural limit of the tennis tower.

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